# 10 Two-Sample Tests

## CONTENTS



## OBJECTIVES

- Compare the means of two independent populations
- Compare the means of two related populations
- Compare the proportions of two independent populations
- Compare the variances of two independent populations

## ▼USING STATISTICS

### *Differing Means for Selling Streaming Media Players at Arlingtons?*

To what extent does the location of products in a store affect sales? At Arlingtons, a general merchandiser that competes with discount and wholesale club retailers, management has been considering this question as part of a general review. Seeking to enhance revenues, managers have decided to create a new sales area at the front of the each Arlingtons store, near the checkout lanes. Management plans to charge product manufacturers a placement fee for placing specific products in this front area, but first need to demonstrate that the area would boost sales.

While some manufacturers refuse to pay such placement fees, Arlingtons has found a willing partner in Pierrsöhn Technologies. Pierrsöhn wants to introduce VLABGo, their new mobile streaming player, and is willing to pay a placement fee to be featured at the front of each Arlingtons store. However, Pierrsöhn management wants reassurance that the front of the store will be worth the placement fee. As the retail operations chief at Arlingtons, you have been asked to negotiate with Pierrsöhn. You propose a test that will involve 20 Arlingtons locations, all with similar storewide sales volumes and shopper demographics. You explain that you will randomly select 10 stores to sell the VLABGo player among other, similar items in the mobile electronics aisle in those Arlingtons stores. For the other 10 stores, you will place the VLABGo players in a special area at the front of the store.

At the end of the one-month test period, the sales of VLABGo players from the two store samples will be recorded and compared. You wonder how you could determine whether the sales in the in-aisle stores are different from the sales in the stores where the VLABGo players appear in the special front area. You also would like to decide if the variability in sales from store to store is different for the two types of sales location. If you can demonstrate a difference in sales, you will have a stronger case for asking for a special front of the store placement fee from Pierrsöhn. What should you do?

Chapter 9 discusses several hypothesis-testing procedures commonly used to test a single sample of data selected from a single population. Hypothesis testing can be extended to **two-sample tests** that compare statistics from samples selected from *two* populations. In the Arlingtons scenario one such test would be "Are the mean VLABGo player monthly sales at the special front location (one population) different from the mean VLABGo player monthly sales at the in-aisle location (a second population)?"

# 10.1 Comparing the Means of Two Independent Populations

Using the correct two-sample test to compare the means of samples selected from each of two independent populations requires first establishing whether the assumption that the variances in the two populations are equal holds. If the assumption holds, you use a *pooled-variance t test*, otherwise you use a *separate variance t test*. Determining whether the assumption that the two variances are equal can be complicated because when you sample from two independent populations, you almost always do not know the standard deviation of either population, as Sections 8.1 and 9.1 note. However, using the sample variances, you can test whether the two population variances are equal using the method that Section 10.4 discusses.

**student TIP**

Whichever population is defined as population 1 in the null and alternative hypotheses must be defined as population 1 in Equation (10.1). Whichever population is defined as population 2 in the null and alternative hypotheses must be defined as population 2 in Equation (10.1).

## Pooled-Variance *t* Test for the Difference Between Two Means Assuming Equal Variances

If you assume that the random samples are independently selected from two populations and that the populations are normally distributed and have equal variances, you can use a **pooled-variance *t* test** to determine whether there is a significant difference between the means. If the populations do not differ greatly from a normal distribution, you can still use the pooled-variance *t* test, especially if the sample sizes are large enough (typically $\geq 30$ for each sample).

Using subscripts to distinguish between the population mean of the first population, $\mu_1$, and the population mean of the second population, $\mu_2$, the null hypothesis of no difference in the means of two independent populations can be stated as

$$H_0: \mu_1 = \mu_2 \quad \text{or} \quad \mu_1 - \mu_2 = 0$$

and the alternative hypothesis, that the means are different, can be stated as

$$H_1: \mu_1 \neq \mu_2 \quad \text{or} \quad \mu_1 - \mu_2 \neq 0$$

To test the null hypothesis, you use the pooled-variance *t* test statistic $t_{STAT}$ shown in Equation (10.1). The pooled-variance *t* test gets its name from the fact that the test statistic pools, or combines, the two sample variances $S_1^2$ and $S_2^2$ to compute $S_p^2$, the best estimate of the variance common to both populations, under the assumption that the two population variances are equal.[1]

[1] When the two sample sizes are equal (i.e., $n_1 = n_2$), the equation for the pooled variance can be simplified to

$$S_p^2 = \frac{S_1^2 + S_2^2}{2}$$

POOLED-VARIANCE *t* TEST FOR THE DIFFERENCE BETWEEN TWO MEANS

$$t_{STAT} = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \qquad \textbf{(10.1)}$$

where

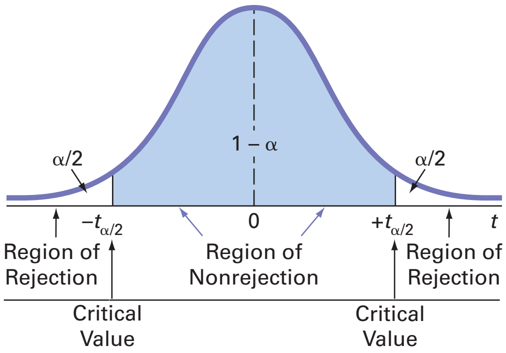
$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)}$$

and
$S_p^2$ = pooled variance
$\overline{X}_1$ = mean of the sample taken from population 1
$S_1^2$ = variance of the sample taken from population 1
$n_1$ = size of the sample taken from population 1
$\overline{X}_2$ = mean of the sample taken from population 2
$S_2^2$ = variance of the sample taken from population 2
$n_2$ = size of the sample taken from population 2

The $t_{STAT}$ test statistic follows a *t* distribution with $n_1 + n_2 - 2$ degrees of freedom.

For a given level of significance, $\alpha$, in a two-tail test, you reject the null hypothesis if the computed $t_{STAT}$ test statistic is greater than the upper-tail critical value from the $t$ distribution or if the computed $t_{STAT}$ test statistic is less than the lower-tail critical value from the $t$ distribution. Figure 10.1 displays the regions of rejection.

**FIGURE 10.1**
Regions of rejection and nonrejection for the pooled-variance $t$ test for the difference between the means (two-tail test)




**student TIP**

When *lower* or *less than* is used in an example, you have a lower-tail test. When *upper* or *more than* is used in an example, you have an upper-tail test. When *different* or *the same as* is used in an example, you have a two-tail test.

In a one-tail test in which the rejection region is in the lower tail, you reject the null hypothesis if the computed $t_{STAT}$ test statistic is less than the lower-tail critical value from the $t$ distribution. In a one-tail test in which the rejection region is in the upper tail, you reject the null hypothesis if the computed $t_{STAT}$ test statistic is greater than the upper-tail critical value from the $t$ distribution.

To demonstrate the pooled-variance $t$ test, return to the Arlingtons scenario on page 383. Using the DCOVA problem-solving approach, you define the business objective as determining whether there is a difference in the mean VLABGo player monthly sales at the special front and in-aisle locations. There are two populations of interest. The first population is the set of all possible VLABGo player monthly sales at the special front location. The second population is the set of all possible VLABGo player monthly sales at the in-aisle location. You collect the data from a sample of 10 Arlingtons stores that have been assigned the special front location and another sample of 10 Arlingtons stores that have been assigned the in-aisle location. You organize the data as Table 10.1 and store the data in **VLABGo**.

**TABLE 10.1**
Comparing VLABGo player Sales from Two Different Locations

| SALES LOCATION | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Special Front** | | | | | **In-Aisle** | | | | |
| 224 | 189 | 248 | 285 | 273 | 192 | 236 | 164 | 154 | 189 |
| 190 | 243 | 215 | 280 | 317 | 220 | 261 | 186 | 219 | 202 |

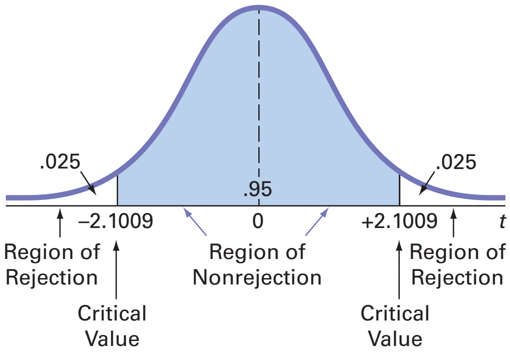
The null and alternative hypotheses are

$$H_0: \mu_1 = \mu_2 \quad \text{or} \quad \mu_1 - \mu_2 = 0$$
$$H_1: \mu_1 \neq \mu_2 \quad \text{or} \quad \mu_1 - \mu_2 \neq 0$$

Assuming that the samples are from normal populations having equal variances, you can use the pooled-variance $t$ test. The $t_{STAT}$ test statistic follows a $t$ distribution with $10 + 10 - 2 = 18$ degrees of freedom. Using an $\alpha = 0.05$ level of significance, you divide the rejection region into the two tails for this two-tail test (i.e., two equal parts of 0.025 each). Table E.3 shows that the critical values for this two-tail test are $+2.1009$ and $-2.1009$. As shown in Figure 10.2 on page 386, the decision rule is

$$\text{Reject } H_0 \text{ if } t_{STAT} > +2.1009$$
$$\text{or if } t_{STAT} < -2.1009;$$
$$\text{otherwise, do not reject } H_0.$$

**FIGURE 10.2**
Two-tail test of hypothesis for the difference between the means at the 0.05 level of significance with 18 degrees of freedom




Using Equation (10.1) on page 384 and the Figure 10.3 descriptive statistics,

$$t_{STAT} = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{9(42.5420)^2 + 9(32.5271)^2}{9 + 9} = 1{,}433.9167$$

resulting in

$$t_{STAT} = \frac{(246.4 - 202.3) - 0.0}{\sqrt{1{,}433.9167\left(\frac{1}{10} + \frac{1}{10}\right)}} = \frac{44.1}{\sqrt{286.7833}} = 2.6041$$

Figure 10.3 shows the Excel, JMP, and Minitab results for the two different sales locations data.

**FIGURE 10.3**
Excel, JMP, and Minitab pooled-variance $t$ test results with confidence interval estimate for the two different sales locations data

| | A | B |
|---|---|---|
| 1 | Pooled-Variance t Test for Differences in Two Means | |
| 2 | (assumes equal population variances) | |
| 3 | Data | |
| 4 | Hypothesized Difference | 0 |
| 5 | Level of Significance | 0.05 |
| 6 | Population 1 Sample | |
| 7 | Sample Size | 10 |
| 8 | Sample Mean | 246.4 |
| 9 | Sample Standard Deviation | 42.5420 |
| 10 | Population 2 Sample | |
| 11 | Sample Size | 10 |
| 12 | Sample Mean | 202.3 |
| 13 | Sample Standard Deviation | 32.5271 |
| 14 | | |
| 15 | Intermediate Calculations | |
| 16 | Population 1 Sample Degrees of Freedom | 9 |
| 17 | Population 2 Sample Degrees of Freedom | 9 |
| 18 | Total Degrees of Freedom | 18 |
| 19 | Pooled Variance | 1433.9167 |
| 20 | Standard Error | 16.9347 |
| 21 | Difference in Sample Means | 44.1 |
| 22 | *t* Test Statistic | 2.6041 |
| 23 | | |
| 24 | Two-Tail Test | |
| 25 | Lower Critical Value | -2.1009 |
| 26 | Upper Critical Value | 2.1009 |
| 27 | *p*-Value | 0.0179 |
| 28 | Reject the null hypothesis | |

| | D | E |
|---|---|---|
| 3 | Confidence Interval Estimate | |
| 4 | for the Difference Between Two Means | |
| 5 | | |
| 6 | Data | |
| 7 | Confidence Level | 95% |
| 8 | | |
| 9 | Intermediate Calculations | |
| 10 | Degrees of Freedom | 18 |
| 11 | *t* Value | 2.1009 |
| 12 | Interval Half Width | 35.5784 |
| 13 | | |
| 14 | Confidence Interval | |
| 15 | Interval Lower Limit | 8.5216 |
| 16 | Interval Upper Limit | 79.6784 |

**Summary Statistics**

| | |
|---|---|
| Sample 1 Mean | 202.3 |
| Sample 1 Standard Deviation | 32.5271 |
| Sample 1 Size | 10 |
| Sample 2 Mean | 246.4 |
| Sample 2 Standard Deviation | 42.542 |
| Sample 2 Size | 10 |
| Pooled Estimate of Standard Deviation | 30.3509 |
| Difference in Sample Means (Mean 2 - Mean 1) | 44.1 |

**Test Results**

| Result | Value |
|---|---|
| Standard Error of the Difference (Mean 2 - Mean 1) | 16.9347 |
| t-score | 2.6041 |
| t Critical Value(s) | +/- 2.1009 |
| Observed Significance (p-value) | 0.0179 |

Reject Null Hypothesis

**Two-Sample T-Test and CI: Special Front, In-Aisle**

**Method**

$\mu_1$: mean of Special Front
$\mu_2$: mean of In-Aisle
Difference: $\mu_1 - \mu_2$

*Equal variances are assumed for this analysis.*

**Descriptive Statistics**

| Sample | N | Mean | StDev | SE Mean |
|---|---|---|---|---|
| Special Front | 10 | 246.4 | 42.5 | 13 |
| In-Aisle | 10 | 202.3 | 32.5 | 10 |

**Estimation for Difference**

| Difference | Pooled StDev | 95% CI for Difference |
|---|---|---|
| 44.1 | 37.9 | (8.5, 79.7) |

**Test**

Null hypothesis $H_0$: $\mu_1 - \mu_2 = 0$
Alternative hypothesis $H_1$: $\mu_1 - \mu_2 \neq 0$

| T-Value | DF | P-Value |
|---|---|---|
| 2.60 | 18 | 0.018 |

Table 10.2 summarizes the results of the pooled-variance $t$ test for the difference between the two sales locations using the calculations on page 386 and the Figure 10.3 results. Based on the conclusions, the special front location generates significantly higher sales. Therefore, as part of the last step of the DCOVA framework, you can offer a justification for charging a placement fee for the special front location.

**TABLE 10.2**
Pooled-variance $t$ test summary for the two sales locations.

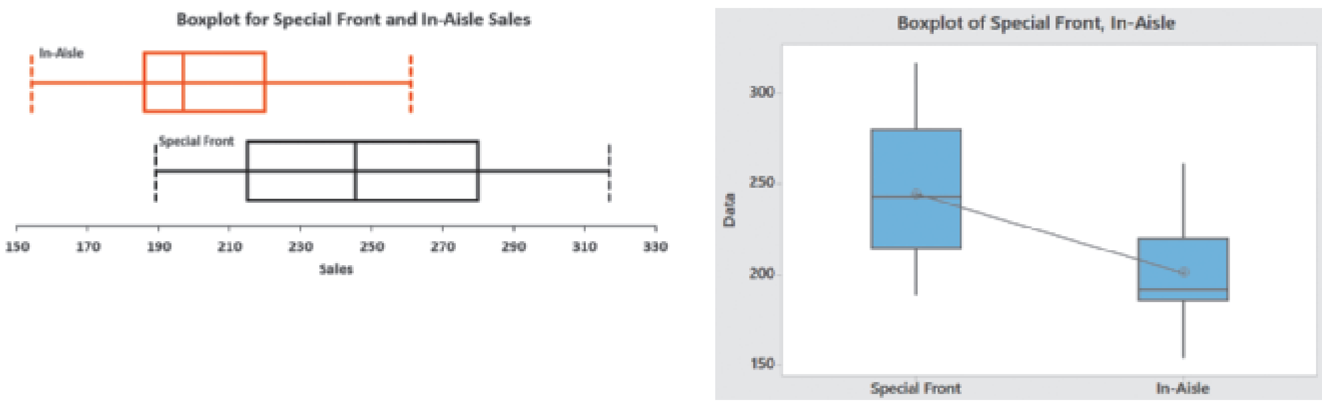
| Result | Conclusions |
|---|---|
| The $t_{STAT} = 2.6041$ is greater than 2.1009. | 1. Reject the null hypothesis $H_0$.<br>2. Conclude that evidence exists that the mean sales are different for the two sales locations. |
| The $t$ test $p$-value $= 0.0179$ is less than the level of significance, $\alpha = 0.05$. | 3. The probability of observing a difference in the two sample means this large or larger is 0.0179. |
| The $t_{STAT}$ is positive. | 4. Conclude that the mean sales are higher for the special front location. |

## Evaluating the Normality Assumption

In testing for the difference between the means, you assume that the populations are normally distributed, with equal variances. For situations in which the two populations have equal variances, the pooled-variance $t$ test is **robust** (i.e., not sensitive) to moderate departures from the assumption of normality, provided that the sample sizes are large. In such situations, you can use the pooled-variance $t$ test without serious effects on its power. However, if you cannot assume that both populations are normally distributed, you have two choices. You can use a nonparametric procedure, such as the Wilcoxon rank sum test (see Section 12.4), that does not depend on the assumption of normality for the two populations, or you can use a normalizing transformation (see reference 4) on each of the values and then use the pooled-variance $t$ test.

To check the assumption of normality in each of the two populations, you can construct a boxplot of the sales for the two display locations shown in Figure 10.4. For these two small samples, there appears to be only slight departure from normality, so the assumption of normality needed for the $t$ test is not seriously violated.

**FIGURE 10.4**
Excel and Minitab boxplots for sales at the special front and in-aisle locations



Example 10.1 provides another application of the pooled-variance $t$ test.

## EXAMPLE 10.1

### Testing for the Difference in the Mean Delivery Times

You and some friends have decided to test the validity of an advertisement by a local pizza restaurant, which says it delivers to the dormitories faster than a local branch of a national chain. Both the local pizza restaurant and national chain are located across the street from your college campus. You define the variable of interest as the delivery time, in minutes, from the time the pizza is ordered to when it is delivered. You collect the data by ordering 10 pizzas from the local pizza restaurant and 10 pizzas from the national chain at different times. You organize and store the data in **PizzaTime**. Table 10.3 shows the delivery times.

**TABLE 10.3**
Delivery Times (in minutes) for a Local Pizza Restaurant and a National Pizza Chain

| Local | | Chain | |
|---|---|---|---|
| 16.8 | 18.1 | 22.0 | 19.5 |
| 11.7 | 14.1 | 15.2 | 17.0 |
| 15.6 | 21.8 | 18.7 | 19.5 |
| 16.7 | 13.9 | 15.6 | 16.5 |
| 17.5 | 20.8 | 20.8 | 24.0 |

At the 0.05 level of significance, is there evidence that the mean delivery time for the local pizza restaurant is less than the mean delivery time for the national pizza chain?

**SOLUTION** Because you want to know whether the mean is *lower* for the local pizza restaurant than for the national pizza chain, you have a one-tail test with the following null and alternative hypotheses:

$H_0: \mu_1 \geq \mu_2$ (The mean delivery time for the local pizza restaurant is equal to or greater than the mean delivery time for the national pizza chain.)

$H_1: \mu_1 < \mu_2$ (The mean delivery time for the local pizza restaurant is less than the mean delivery time for the national pizza chain.)

Figure 10.5 displays the results for the pooled-variance $t$ test for these data.

**FIGURE 10.5**
Excel and JMP pooled-variance $t$ test results for the pizza delivery time data

*JMP reports the test statistic and critical value as positive values.*

| | A | B |
|---|---|---|
| 1 | Pooled-Variance t Test for Differences in Two Means | |
| 2 | (assumes equal population variances) | |
| 3 | Data | |
| 4 | Hypothesized Difference | 0 |
| 5 | Level of Significance | 0.05 |
| 6 | Population 1 Sample | |
| 7 | Sample Size | 10 |
| 8 | Sample Mean | 16.7 |
| 9 | Sample Standard Deviation | 3.0955 |
| 10 | Population 2 Sample | |
| 11 | Sample Size | 10 |
| 12 | Sample Mean | 18.88 |
| 13 | Sample Standard Deviation | 2.8662 |
| 14 | | |
| 15 | Intermediate Calculations | |
| 16 | Population 1 Sample Degrees of Freedom | 9 |
| 17 | Population 2 Sample Degrees of Freedom | 9 |
| 18 | Total Degrees of Freedom | 18 |
| 19 | Pooled Variance | 8.8986 |
| 20 | Standard Error | 1.3341 |
| 21 | Difference in Sample Means | -2.18 |
| 22 | *t* Test Statistic | -1.6341 |
| 23 | | |
| 24 | Lower-Tail Test | |
| 25 | Lower Critical Value | -1.7341 |
| 26 | *p*-Value | 0.0598 |
| 27 | Do not reject the null hypothesis | |

**Test Inputs**

| | |
|---|---|
| Hypothesized Difference Mean 2 - Mean 1) | 0 |
| Sample 1 Mean | 16.7 |
| Sample 1 Standard Deviation | 3.0955 |
| Sample 1 Size | 10 |
| Sample 2 Mean | 18.88 |
| Sample 2 Standard Deviation | 2.8662 |
| Sample 2 Size | 10 |
| Significance Level (alpha) | 0.05 |

**Test Results**

| Result | Value |
|---|---|
| Observed Difference (Mean 2 - Mean 1) | 2.18 |
| Standard Error of the Difference (Mean 2 - Mean 1) | 1.3341 |
| t-score | 1.6341 |
| t critical values | 1.7341 |
| Observed Significance (p-value) | 0.0598 |

Fail to Reject Null Hypothesis

▶(*continued*)

To illustrate the computations, using Equation (10.1) on page 384,

$$t_{STAT} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)}$$

$$= \frac{9(3.0955)^2 + 9(2.8662)^2}{9 + 9} = 8.8986$$

Therefore,

$$t_{STAT} = \frac{(16.7 - 18.88) - 0.0}{\sqrt{8.8986\left(\frac{1}{10} + \frac{1}{10}\right)}} = \frac{-2.18}{\sqrt{1.7797}} = -1.6341$$

Table 10.4 summarizes the results of the pooled-variance $t$ test for the pizza delivery data using the calculations above and Figure 10.5 results. Based on the conclusions, the local branch of the national chain and a local pizza restaurant have similar delivery times. Therefore, as part of the last step of the DCOVA framework, you and your friends exclude delivery time as a decision criteria when choosing from which store to order pizza.

**TABLE 10.4**
Pooled-variance $t$ test summary for the delivery times for the two pizza restaurants

| Result | Conclusions |
|---|---|
| The $t_{STAT} = -1.6341$ is greater than $-1.7341$.<br>The $t$ test $p$-value $= 0.0598$ is greater than the level of significance, $\alpha = 0.05$. | 1. Do not reject the null hypothesis $H_0$.<br>2. Conclude that insufficient evidence exists that the mean delivery time is lower for the local restaurant than for the branch of the national chain.<br>3. There is a probability of 0.0598 that $t_{STAT} < -1.6341$. |

## Confidence Interval Estimate for the Difference Between Two Means

Instead of, or in addition to, testing for the difference between the means of two independent populations, you can use Equation (10.2) to develop a confidence interval estimate of the difference in the means.

CONFIDENCE INTERVAL ESTIMATE FOR THE DIFFERENCE BETWEEN THE MEANS OF TWO INDEPENDENT POPULATIONS

$$\left(\bar{X}_1 - \bar{X}_2\right) \pm t_{\alpha/2}\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \qquad \textbf{(10.2)}$$

or

$$\left(\bar{X}_1 - \bar{X}_2\right) - t_{\alpha/2}\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \le \mu_1 - \mu_2 \le \left(\bar{X}_1 - \bar{X}_2\right) + t_{\alpha/2}\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

where $t_{\alpha/2}$ is the critical value of the $t$ distribution, with $n_1 + n_2 - 2$ degrees of freedom, for an area of $\alpha/2$ in the upper tail.

For the sample statistics pertaining to the two locations reported in Figure 10.3 on page 386, using 95% confidence, and Equation (10.2),

$$\bar{X}_1 = 246.4,\ n_1 = 10,\ \bar{X}_2 = 202.3,\ n_2 = 10,\ S_p^2 = 1{,}433.9167, \text{ and with } 10 + 10 - 2$$
$$= 18 \text{ degrees of freedom, } t_{0.025} = 2.1009$$

$$(246.4 - 202.3) \pm (2.1009)\sqrt{1{,}433.9167\left(\frac{1}{10} + \frac{1}{10}\right)}$$
$$44.10 \pm (2.1009)(16.9347)$$
$$44.10 \pm 35.5784$$
$$8.5216 \le \mu_1 - \mu_2 \le 79.6784$$

Therefore, you are 95% confident that the difference in mean sales between the special front and in-aisle locations is between 8.5216 and 79.6784 VLABGo players sold. In other words, you can estimate, with 95% confidence, that the special front location has mean sales of between 8.5216 and 79.6784 more VLABGo players than the in-aisle location. From a hypothesis-testing perspective, using a two-tail test at the 0.05 level of significance, because the interval does not include zero, you reject the null hypothesis of no difference between the means of the two populations.

## Separate-Variance *t* Test for the Difference Between Two Means, Assuming Unequal Variances

If you can assume that the two independent populations are normally distributed but cannot assume that they have equal variances, you cannot pool the two sample variances into the common estimate $S_p^2$ and therefore cannot use the pooled-variance $t$ test. Instead, you use the **separate-variance *t* test** developed by Satterthwaite that uses the two separate sample variances (see reference 3 and the online topic **Separate-Variance *t* Test Calculations**).

Figure 10.6 displays the separate-variance $t$ test results for the two different sales locations data. Observe that the test statistic $t_{STAT} = 2.6041$ and the $p$-value is $0.019 < 0.05$. The results for the separate-variance $t$ test are nearly the same as those of the pooled-variance $t$ test. The assumption of equality of population variances had no appreciable effect on the results.

Sometimes, the results from the pooled-variance and separate-variance $t$ tests conflict because the assumption of equal variances is violated. Therefore, you must evaluate the assumptions and use those results as a guide in selecting a test procedure. In Section 10.4, the

**FIGURE 10.6**

Excel, JMP, and Minitab separate-variance *t* test results for the two different sales locations data

| | A | B |
|---|---|---|
| 1 | Separate-Variances *t* Test | |
| 2 | (assumes unequal population variances) | |
| 3 | Data | |
| 4 | Hypothesized Difference | 0 |
| 5 | Level of Significance | 0.05 |
| 6 | Population 1 Sample | |
| 7 | Sample Size | 10 |
| 8 | Sample Mean | 246.4 |
| 9 | Sample Standard Deviation | 42.5420 |
| 10 | Population 2 Sample | |
| 11 | Sample Size | 10 |
| 12 | Sample Mean | 202.3 |
| 13 | Sample Standard Deviation | 32.5271 |
| 14 | | |
| 15 | Intermediate Calculations | |
| 16 | Numerator of Degrees of Freedom | 82244.6803 |
| 17 | Denominator of Degrees of Freedom | 4881.1600 |
| 18 | Total Degrees of Freedom | 16.8425 |
| 19 | Degrees of Freedom | 16 |
| 20 | Standard Error | 16.9347 |
| 21 | Difference in Sample Means | 44.1000 |
| 22 | Separate-Variance *t* Test Statistic | 2.6041 |
| 23 | | |
| 24 | Two-Tail Test | |
| 25 | Lower Critical Value | -2.1199 |
| 26 | Upper Critical Value | 2.1199 |
| 27 | *p*-Value | 0.0192 |
| 28 | Reject the null hypothesis | |

**Summary Statistics**

| | |
|---|---|
| Sample 1 Mean | 202.3 |
| Sample 1 Standard Deviation | 32.5271 |
| Sample 1 Size | 10 |
| Sample 2 Mean | 246.4 |
| Sample 2 Standard Deviation | 42.542 |
| Sample 2 Size | 10 |
| Pooled Estimate of Standard Deviation | 30.3509 |
| Difference in Sample Means (Mean 2 - Mean 1) | 44.1 |

**Test Results**

| Result | Value |
|---|---|
| Standard Error of the Difference (Mean 2 - Mean 1) | 16.9347 |
| t-score | 2.6041 |
| t Critical Value(s) | +/- 2.1113 |
| Observed Significance (p-value) | 0.0186 |

**Two-Sample T-Test and CI: Special Front, In-Aisle**

**Method**

$\mu_1$: mean of Special Front

$\mu_2$: mean of In-Aisle

Difference: $\mu_1 - \mu_2$

*Equal variances are not assumed for this analysis.*

**Descriptive Statistics**

| Sample | N | Mean | StDev | SE Mean |
|---|---|---|---|---|
| Special Front | 10 | 246.4 | 42.5 | 13 |
| In-Aisle | 10 | 202.3 | 32.5 | 10 |

**Estimation for Difference**

| Difference | 95% CI for Difference |
|---|---|
| 44.1 | (8.2, 80.0) |

**Test**

Null hypothesis $H_0: \mu_1 - \mu_2 = 0$

Alternative hypothesis $H_1: \mu_1 - \mu_2 \ne 0$

| T-Value | DF | P-Value |
|---|---|---|
| 2.60 | 16 | 0.019 |